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Mathematics and Muslims

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The Islamic world made significant contributions in mathematical sciences from the 8th century onwards. Muslim mathematicians drew upon and blended together the mathematical advances of Greece and India. The Islamic prohibition on depicting the human form led to the extensive use of complex geometric patterns to decorate buildings resulting in mathematics practiced as art.

The House of Wisdom (Bait-ul-Hikmah) that was set up in Baghdad around 810 AD started translating the major mathematical works of the Greeks and Indians into the Arabic language. The genius mathematician Mohammed Al-Khwarizmi was an early Rector of the House of Wisdom in the 9th Century.

Muhammad ibn Musa al-Khwarizmi (d. 850), whose name was Latinized as Algoritmi, made huge contributions to mathematics and laid the basis for innovation in algebra and trigonometry. His magnum opus *The Compendious Book on Calculation by Completion and Balancing* (Al-Kitab al-mukhtaṣar fi ḥisab al-jabr wal-muqabala) is a mathematical book written around 830 AD. The term Algebra is derived from the name of one of the basic operations with equations Al-Khwarizmi described in his book; al-jabr, which means restoration, refers to adding a number to both sides of the equation to consolidate or cancel terms. The book was translated into Latin as *Liber algebrae et almucabala* by Robert of Chester (Segovia, 1145 AD) and also by Gerard of Cremona. The book provided an exhaustive account of solving polynomial equations up to the second degree, and discussed the fundamental methods of reduction and balancing, referring to the transposition of terms to the other side of an equation, that is, the cancellation of like terms on opposite sides of the equation (Boyer, 1991). Al-Khwarizmi also solved linear and quadratic equations.

The beginnings of algebra were a revolutionary move away from the Greek concept of mathematics, which was essentially geometrical. Algebra was a unifying theory that allowed rational numbers, irrational numbers, and geometrical magnitudes to be treated as algebraic objects. This not only

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provided mathematics a whole new direction much broader than that which had existed before, it also provided an instrument for future development of the subject. Al-Khwarizmi introduced the fundamental algebraic methods of reduction and balancing and provided an exhaustive account of solving polynomial equations up to the second degree. In this way, he contributed to the creation of a powerful abstract mathematical language that is still used throughout the world today, a language that allowed a better way of analyzing mathematical problems.

Al-Khwarizmi's second major work *On the Calculation with Hindu Numerals* (Kitab al-Jam' wat-Tafriq bi-Hisab al-Hind) was written about 825 AD, and served to spread the Hindu-Arabic numeral system throughout the Middle East and Europe. This was translated into Latin as *Algoritmi de numero Indorum*. Al-Khwarizmi, whose name was rendered in Latin to Algoritmi, led to the term algorithm. His most important contribution to mathematics was his strong advocacy of the Hindu-Arabic numeral system (1-9 and 0), developed in Indian mathematics, which he recognized as having the power and efficiency needed to revolutionize mathematics, and which was soon adopted by the entire world. His book, *Zij al-Sindhind* also contained tables for the trigonometric functions of sines and cosines. A related treatise on spherical trigonometry is also attributed to him.

Mohammed Al-Karaji (1029 AD) pushed the frontiers of algebra still further by freeing it from its geometrical heritage and introducing the theory of algebraic calculus. Al-Karaji was the first to use the method of proof by mathematical induction to prove his results, by showing that the first statement in an infinite sequence of statements is true, and then proving that, if any one statement in the sequence is true, then so is the next one. He used mathematical induction to prove the binomial theorem. A binomial is a simple type of algebraic expression that has just two terms which are operated on only by addition, subtraction, multiplication and positive whole-number exponents, such as $(x+y)$. The coefficients needed when a binomial is expanded form a symmetrical triangle; this is usually referred to as Pascal's Triangle after the 17th Century mathematician Pascal (1662 AD), even though many other mathematicians had studied it centuries before him including Al-Karaji.

Omar Khayyam (best known as the writer of the Rubaiyat) generalized Indian methods for extracting square and cube roots to include fourth, fifth and higher roots in the early 12th Century.

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He carried out a systematic analysis of cubic problems, revealing that there were actually several different sorts of cubic equations. He wrote the influential Treatise on Demonstration of Problems of Algebra (AD 1070), which laid down the principles of algebra that were eventually transmitted to Europe. In particular, he derived general methods for solving cubic equations and even higher orders. In the Treatise, Khayyam wrote on the triangular array of binomial coefficients, what would become known as Pascal's triangle. In AD 1077, Khayyam wrote Sharh ma ashkala min musadarat kitab Uqlidis (Explanations of the Difficulties in the Postulates of Euclid) published in English as On the Difficulties of Euclid's Definitions.

Nasir Al-Din Al-Tusi (1274 AD) was the first to treat trigonometry as a separate mathematical discipline, distinct from astronomy. Building on earlier work by Greek and Indian works on the sine function, he gave the first extensive exposition of spherical trigonometry, including listing the six distinct cases of a right triangle in spherical trigonometry. One of his major mathematical contributions was the formulation of the famous law of sines for plane triangles, $a/\sin A = b/\sin B = c/\sin C$, although the sine law for spherical triangles had been discovered earlier by the 10th Century Persians Abul Wafa Buzjani and Abu Nasr Mansur.

Thabit ibn Qurra (901 AD) developed a general formula by which amicable numbers could be derived. Amicable numbers are pairs of numbers for which the sum of the divisors of one number equals the other number; for example, the proper divisors of 220 are 1, 2, 4, 5, 10, 11, 20, 22, 44, 55 and 110, of which the sum is 284; and the proper divisors of 284 are 1, 2, 4, 71, and 142, of which the sum is 220. This formula was later re-discovered much later by both Fermat (1665 AD) and Descartes (1650 AD).

In addition to his monumental works on optics and physics Ibn al-Haytham (also known as Alhazen-1040 AD) , established the beginnings of the link between Algebra and Geometry, and devised what is now known as Alhazen's problem. This eventually led al-Haytham to derive a formula for the sum of fourth powers, where previously only the formulas for the sums of squares and cubes had been stated. His method can be readily generalized to find the formula for the sum of any integral powers. He used his result on sums of integral powers to perform what would now be

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called integration, where the formulas for the sums of integral squares and fourth powers allowed him to calculate the volume of a paraboloid.

Abu Kamil (d. 930 AD), wrote the Book on Algebra. The book dealt with solutions of quadratic equations, application of algebra to geometry and Diophantine equations. He is considered to be the first mathematician to systematically use and accept irrational numbers as solutions and coefficients to equations. His mathematical techniques were later adopted by the Italian mathematician Fibonacci (1250 AD), affording him the opportunity to play an important role in introducing Algebra to Europe. He was the first Muslim mathematician to work easily with algebraic equations with powers higher than x^2 (up to x^8) and solved sets of non-linear simultaneous equations with three unknown variables. He wrote all problems rhetorically, and some of his books lacked mathematical notation. For example, he uses the Arabic expression *mal mal shay* (square-square-thing) for X^5 (i.e., $X^2.X^2.X$). He improved the work of al-Khwarizmi and applied algebraic methods to geometry. His research also included quadratic equations, multiplication and division of algebraic quantities, addition and subtraction of radicals.

From this brief discussion, we can safely conclude that the Muslim made great contributions in the advancement of Algebra and Mathematics, paving the way for making Mathematics as basis for all scientific measurements.

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